Kronecker – Weber Theorem

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Number Field

$\mathbb{Q}(a, b)$

$(x^3 - 1) = 0$

$a + 2b$

σ

$\mathbb{C}$

$b + 2a$

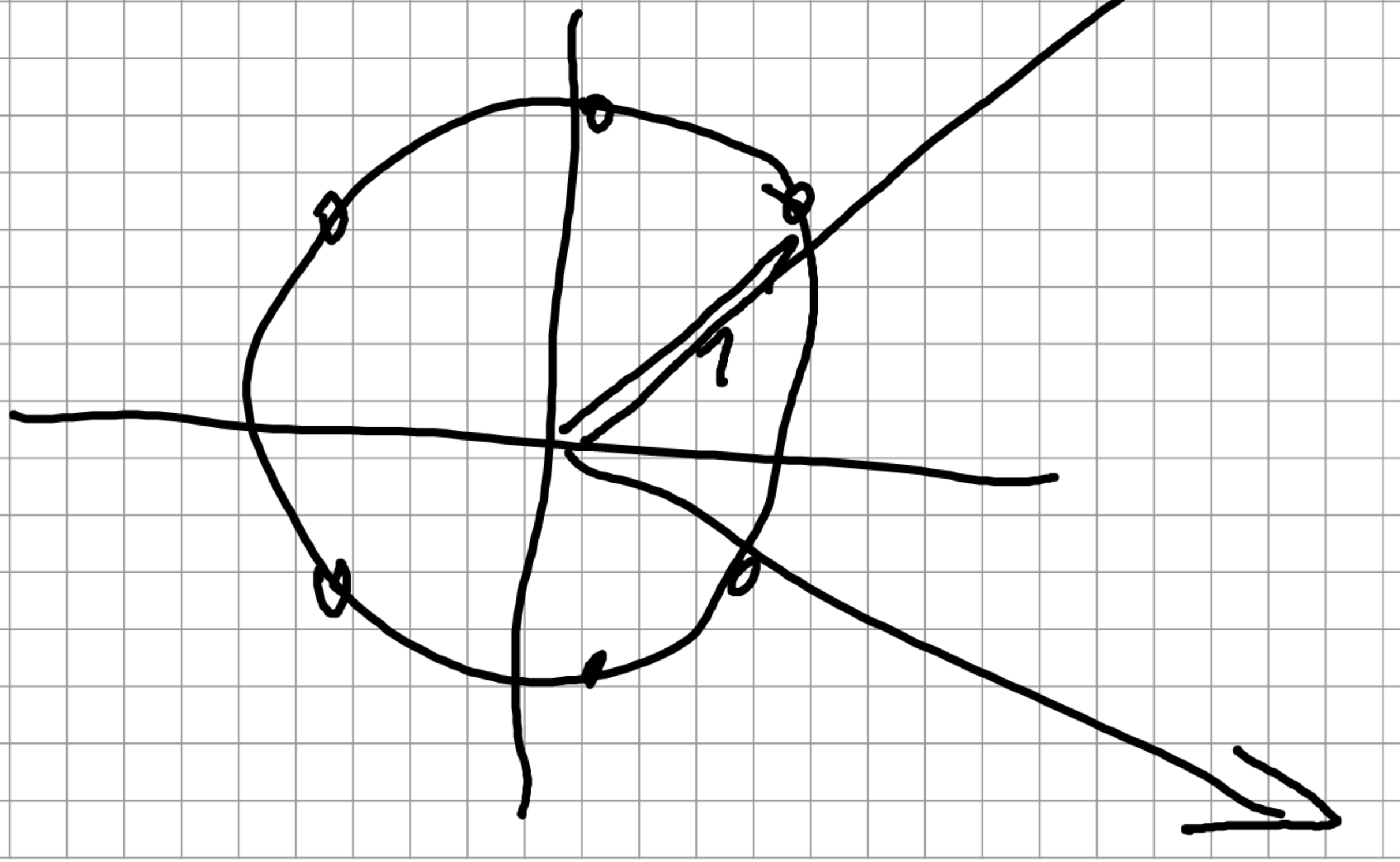
$\sigma_i : \zeta_i \mapsto \zeta_b$ $\quad \{\sigma_i\}_{i \in I}$

Galois group

$\sigma_a \sigma_b (x) = \sigma_b \sigma_a (x)$

Cyclotomic

$\mathbb{Q}(\zeta)$ $\quad \zeta^n = 1$



↗ reduce degree $\longrightarrow p^m$

↗ reduce $\delta \longrightarrow p^k$

↗ $\forall p$

$$\deg(L) = \prod p_i^{e_i}$$

$$\mathrm{Gal}(L) \cong \bigotimes G_i$$

$$L \simeq \bigotimes \mathbb{Q}(\zeta_i)$$

$$\deg(L_i) = p_i^{e_i}$$

$$\mathbb{Q}(\zeta_i)$$

$$\delta = p^k \prod q_i^{k_i}$$

$q_i \to q$

$\to L(\mathcal{F}_q)$

L

$\delta = p^k \prod_{i \neq 1} q_i^{k_i}$

$\deg = p^{\cdot}$ $(\mathcal{F}_q)$

$\Leftarrow$ $Q(\mathcal{F}_q)$